3.10–3.12 Test Review Key

Name________________________ **Date** __________ **Period** ________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the fundamental identities to find the value of the trigonometric function.

1) Find $\cos\theta$ if $\sin\theta = -\frac{12}{13}$ and $\tan\theta > 0$. 1) __________

$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{-12}{13}$ so, $y = -12$, $r = 13$

Find x. $x^2 + y^2 = r^2$

so, $x^2 + (-12)^2 = 13^2$ $x = \pm 5$, since $\tan\theta > 0$ $x = -5$ $\boxed{\cos\theta = \frac{-5}{13}}$

2) Find $\tan\theta$ if $\cos\theta = \frac{1}{4}$ and $\sin\theta < 0$. 2) __________

$\tan\theta = \frac{y}{x}$, $\cos\theta = \frac{1}{4}$ so, $x = 1$, $r = 4$

Find y. $x^2 + y^2 = r^2$

$(1)^2 + y^2 = (4)^2$ $y = \pm\sqrt{15}$, $\sin\theta < 0$ so, $\boxed{\tan\theta = -\sqrt{15}}$

Use basic identities to simplify the expression.

3) $\frac{\csc\theta \cot\theta}{\sec\theta}$ 3) __________

$$\frac{\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\frac{\cos\theta}{\sin^2\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin^2\theta} \cdot \frac{\cos\theta}{1} = \frac{\cos^2\theta}{\sin^2\theta} = \boxed{\cot^2\theta}$$

4) $\frac{\cos^2\theta}{\sin^2\theta} + \csc\theta \sin\theta$ 4) __________

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{1} = \cot^2\theta + 1 = \boxed{\csc^2\theta}$$

5) $\frac{\tan\theta}{\sec\theta}$ $\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} = \boxed{\sin\theta}$ 5) __________

Simplify the expression.

6) sec (− x) cos (− x)

6) ______________

$(\sec x)(\cos x) = \frac{1}{\cos x}\cdot\frac{\cos x}{1} = \boxed{1}$

7) cos x + sin x tan x

7) ______________

$\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \left(\frac{\cos x}{1}\right) + \frac{\sin^2 x}{\cos x}$

$\frac{\cos x}{\cos x}\cdot\frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$

8) $\frac{\cos x}{\tan^2 x} - \frac{\cos x}{\sin^2 x}$

8) ______________

$\frac{\cos x}{\frac{\sin^2 x}{\cos^2 x}} - \frac{\cos x}{\sin^2 x} = \frac{\cos x}{1}\cdot\frac{\cos^2 x}{\sin^2 x} - \frac{\cos x}{\sin^2 x} = \frac{\cos^3 x - \cos x}{\sin^2 x} = \frac{\cos x(\cos^2 x - 1)}{\sin^2 x}$

$= \frac{\cos x(-\sin^2 x)}{\sin^2 x} = \boxed{-\cos x}$

9) $\frac{\sec x}{\sin x} - \frac{\cos x}{\sin x}$

9) ______________

$\frac{\sec x - \cos x}{\sin x} = \frac{\frac{1}{\cos x} - \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right)}{\sin x} = \frac{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}}{\sin x} = \frac{\frac{1-\cos^2 x}{\cos x}}{\sin x} = \frac{\sin^2 x}{\cos x}\cdot\frac{1}{\sin x} = \frac{\sin x}{\cos x}$

$\sin^2 x$

$\boxed{= \tan x}$

Find all solutions in the interval [0, 2π).

10) $2 \sin^2 x = \sin x$

10) ______________

Factor

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \qquad 2\sin x - 1 = 0$

$x = 0, \pi \qquad \sin x = 1/2$

$x = \pi/6, 5\pi/6$

11) $4 \sin^2 x - 4 \sin x + 1 = 0$

11) ______________

Factor

$(2\sin x - 1)^2 = 0$

$\sin x = 1/2$

$\boxed{x = \pi/6 + 5\pi/6}$

Sum + difference Identities

Find an exact value.

12) sin 15°

12) ____________

$$\sin(45°-30°) = \sin 45°\cdot\cos 30° - \sin 30°\cdot\cos 45°$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

13) tan 15°

13) ____________ ok to stop here. rationalize

$$\tan(45°-30°) = \frac{\tan 45° - \tan 30°}{1+\tan 45°\cdot\tan 30°} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}\cdot\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$$
$$= \frac{3-\sqrt{3}-\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} = \boxed{2-\sqrt{3}}$$

14) $\cos\frac{\pi}{12} = \cos\left(\frac{3\pi}{12}-\frac{2\pi}{12}\right)$

14) ____________

$$\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \cos\pi/4\cdot\cos\pi/6 + \sin\pi/4\cdot\sin\pi/6$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

15) $\tan\frac{7\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

15) ____________ ok with this! to rationalize

$$= \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \frac{\tan\pi/4 + \tan\pi/3}{1-\tan\pi/4\cdot\tan\pi/3} = \boxed{\frac{1+\sqrt{3}}{1-\sqrt{3}}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+\sqrt{3}+\sqrt{3}+3}{1-3}$$
$$= \frac{4+2\sqrt{3}}{-2} = \boxed{-2-\sqrt{3}}$$

Write the expression as the sine, cosine, or tangent of an angle.

16) cos 133° cos 58° + sin 133° sin 58°

16) ____________

$$\cos(133°-58°) = \boxed{\cos 75°}$$

17) sin 8x cos x − cos 8x sin x

17) ____________

$$\sin(8x-x) = \boxed{\sin 7x}$$

Prove the identity.

18) $\cos 4x = 1 - 2\sin^2 2x$ 18) ______

Rewrite:

$\cos(2x+2x) = \cos 2x \cdot \cos 2x - \sin 2x \cdot \sin 2x$

use sum identity

$= \cos^2 2x - \sin^2 2x$

Pyth. Ident.

$= (1-\sin^2 2x) - \sin^2 2x = 1 - 2\sin^2 2x$ ✓

Rewrite with only sin x and cos x.

19) $\sin 2x - \cos 3x$ 19) ______

$2\sin x\cos x - \cos(2x+x) = 2\sin x\cos x + (\cos 2x\cos x - \sin 2x\sin x)$

$= 2\sin x\cos x + (\cos^2 x - \sin^2 x)\cos x - (2\sin^2 x\cos x)$

$= 2\sin x\cos x + \cos^3 x - \sin^2 x\cos x - 2\sin^2 x\cos x$

$= 2\sin x\cos x + \cos^3 x - 3\sin^2 x\cos x$

two other possible answers.

Find all solutions to the equation in the interval $[0, 2\pi)$.

20) $\cos 2x - \cos x = 0$ 20) ______

double angle ident.

$(2\cos^2 x - 1) - \cos x = 0$

Factor

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$

$\cos x = -1/2$

$x = 2\pi/3, 4\pi/3$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0$

21) $\sin 2x = -\sin x$ 21) ______

$(\sin 2x) + \sin x = 0$

double angle ident.

$(2\sin x\cos x) + \sin x = 0$

Factor

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$2\cos x + 1 = 0$

$\cos x = -1/2$

$x = \frac{2\pi}{3}$ & $\frac{4\pi}{3}$